

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT3

Branch: B. Tech (All)

Semester : 2

Date : 23/10/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

a) The interval of convergence of the logarithmic series

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty \text{ is}$$

(A) $-1 < x \leq 1$ (B) $-1 < x < 2$ (C) $-\infty < x < \infty$ (D) $-1 \leq x \leq 1$

b) The infinite series $1 + r + r^2 + \dots + r^{n-1}$ is divergent if

(A) $|r| < 1$ (B) $|r| > 1$ (C) $r \geq 1$ (D) $r = -1$

c) $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ is equal to

(A) $\frac{1}{16}$ (B) $\frac{1}{32}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{32}$

d) Let $f(b)$ be an odd function in the interval $\left[-\frac{T}{2}, \frac{T}{2}\right]$ with a period T , then

$$F(x) = \int_a^x f(t) \, dt \text{ is}$$

(A) periodic (B) non-periodic (C) periodic with period $2T$
(D) periodic with period $4T$

e) If f is continuous, decreasing function on $[1, \infty)$ and $\lim_{x \rightarrow \infty} f(x) = 0$, then

$$\int_1^{\infty} f(x) \, dx \text{ is convergent.}$$

(A) True (B) False

f) $\sqrt[4]{4.5} = \underline{\hspace{2cm}}$

(A) $\frac{\sqrt{\pi}}{16}$ (B) $\frac{105\sqrt{\pi}}{16}$ (C) $\frac{5\sqrt{\pi}}{16}$ (D) none of these



- g) Duplication formula: $\sqrt[n]{n + \frac{1}{2}} = \underline{\hspace{2cm}}$
 (A) $\frac{\sqrt{\pi} \sqrt{n}}{2^{2n-1}}$ (B) $\frac{\sqrt{\pi} \sqrt{2n}}{2^{n-1}}$ (C) $\frac{\sqrt{\pi} \sqrt{2n}}{2^{2n-1}}$ (D) $\frac{\sqrt{\pi} \sqrt{n}}{2^{n-1}}$
- h) If the two tangents at the point are real and distinct the double point is called
 (A) a node (B) a cusp (C) a conjugate point (D) none of these
- i) $\int_0^1 \int_0^x e^x dx dy$ is equal to
 (A) -1 (B) 1 (C) e (D) e^{-1}
- j) The transformations $x + y = u, y = uv$ transform the area element $dy dx$ into $|J| du dv$, where $|J|$ is equal to
 (A) 1 (B) u (C) -1 (D) none of these
- k) $\int_0^2 \int_1^3 \int_1^2 xy^2z dz dy dx = \underline{\hspace{2cm}}$
 (A) 62 (B) 26 (C) 24 (D) 42
- l) The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x \log\left(\frac{d^2y}{dx^2}\right)$ is
 (A) 1 (B) 2 (C) 3 (D) none of these
- m) If $\frac{dy}{dx} + \frac{1}{y\sqrt{1-x^2}} = 0$, then which of the following statements is true?
 (A) $y + \sin^{-1} x = 0$ (B) $y^2 + 2\sin^{-1} x = c$ (C) $x + \sin^{-1} y = c$ (D) $y = k$
- n) The homogeneous differential equation $f_1(x, y)dx + f_2(x, y)dy = 0$ can be reduced to a differential equation in which the variables are separated, by the substitution
 (A) $y = vx$ (B) $x + y = v$ (C) $xy = v$ (D) $x - y = v$

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

- a) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$ is (i) convergent if $p > 1$ and (ii) divergent if $p \leq 1$.

(5)

- b) Using reduction formula evaluate $\int_0^{\frac{1}{2}} x^3 \sqrt{1-4x^2} dx$.

(5)

- c) Prove that $\int_0^2 x^4 (8-x^3)^{-\frac{1}{3}} dx = \frac{16}{3} B\left(\frac{5}{3}, \frac{2}{3}\right)$.

(4)

Q-3

Attempt all questions

(14)

- a) Evaluate: $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx$

(5)

- b) Using reduction formula prove that $\int_0^a x^5 (2a^2 - x^2)^{-3} dx = \frac{1}{2} \left(\log 2 - \frac{1}{2}\right)$.

(5)



c) Discuss the convergence of $\sum \frac{\sqrt{n+1} - \sqrt{n}}{n}$. (4)

Q-4 Attempt all questions (14)

a) Change the order of integration in $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx dy$ and evaluate it. (5)

b) Test for convergence the series $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \dots (x > 0)$ by ratio test. (5)

c) Solve: $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$ (4)

Q-5 Attempt all questions (14)

a) Solve: $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$ (5)

b) Evaluate the double integral $\iint_R (x^2 + y^2) dx dy$, where R is the square bounded by lines $y = x, y = -x, x - y = 2, x + y = 2$ using transformations, $u = x + y$ and $v = x - y$. (5)

c) Using reduction formula, evaluate $\int_0^{\frac{\pi}{6}} \cos^6 3\theta \sin^2 6\theta d\theta$. (4)

Q-6 Attempt all questions (14)

a) Prove that $\int_{-\infty}^{\infty} e^{-k^2 x^2} dx = \frac{\sqrt{\pi}}{k}$. (5)

b) Solve: $(x^2 - y^2) dy = 2xy dx$ (5)

c) Evaluate: $\int_0^{\frac{\pi}{2}} \int_0^{a \sin \theta} \int_0^{\frac{a^2 - r^2}{a}} r dr d\theta dz$ (4)

Q-7 Attempt all questions (14)

a) Find the asymptotes of the curve $y^3 - x^2(6 - x) = 0$. (5)

b) Find the area of the region outside the circle $r = 2$ and inside the lemniscate $r^2 = 8 \cos 2\theta$. (5)

c) Investigate the convergence of $\int_2^5 \frac{1}{\sqrt{(x-2)}} dx$. (4)

Q-8 Attempt all questions (14)

a) Evaluate: $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$ (5)

b) Trace the curve $r = a(1 + \cos \theta)$. (5)

c) Find the length of the arc of the Catenary $y = c \cosh\left(\frac{x}{c}\right)$ measured from the vertex $(0, c)$ to any point on the Catenary. (4)

